Spreading Processes

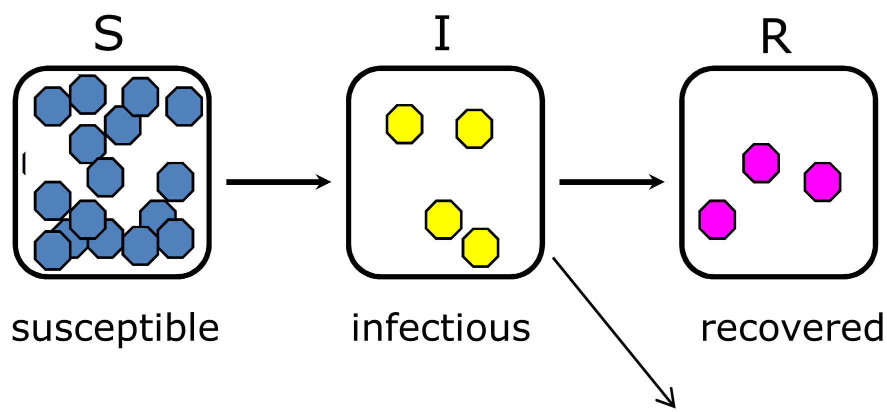
Modeling Infectious Disease Dynamics with Networks

What is epidemiology?

- **Terms**
  - Susceptible
  - Infected
  - Epidemic

- **Questions asked:**
  - will an epidemic occur?
  - what is the typical size of an outbreak?
  - what determines the probability of an epidemic?
  - How do we control the spread?

Compartmental models

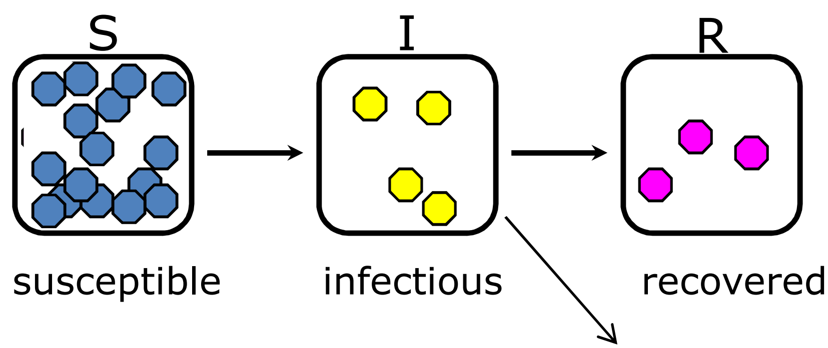


$$\frac{dS}{dt} = -\beta IS$$

$$\frac{dI}{dt} = \beta IS - \gamma I - \alpha I$$

$$\frac{dR}{dt} = \gamma I$$

Compartmental models

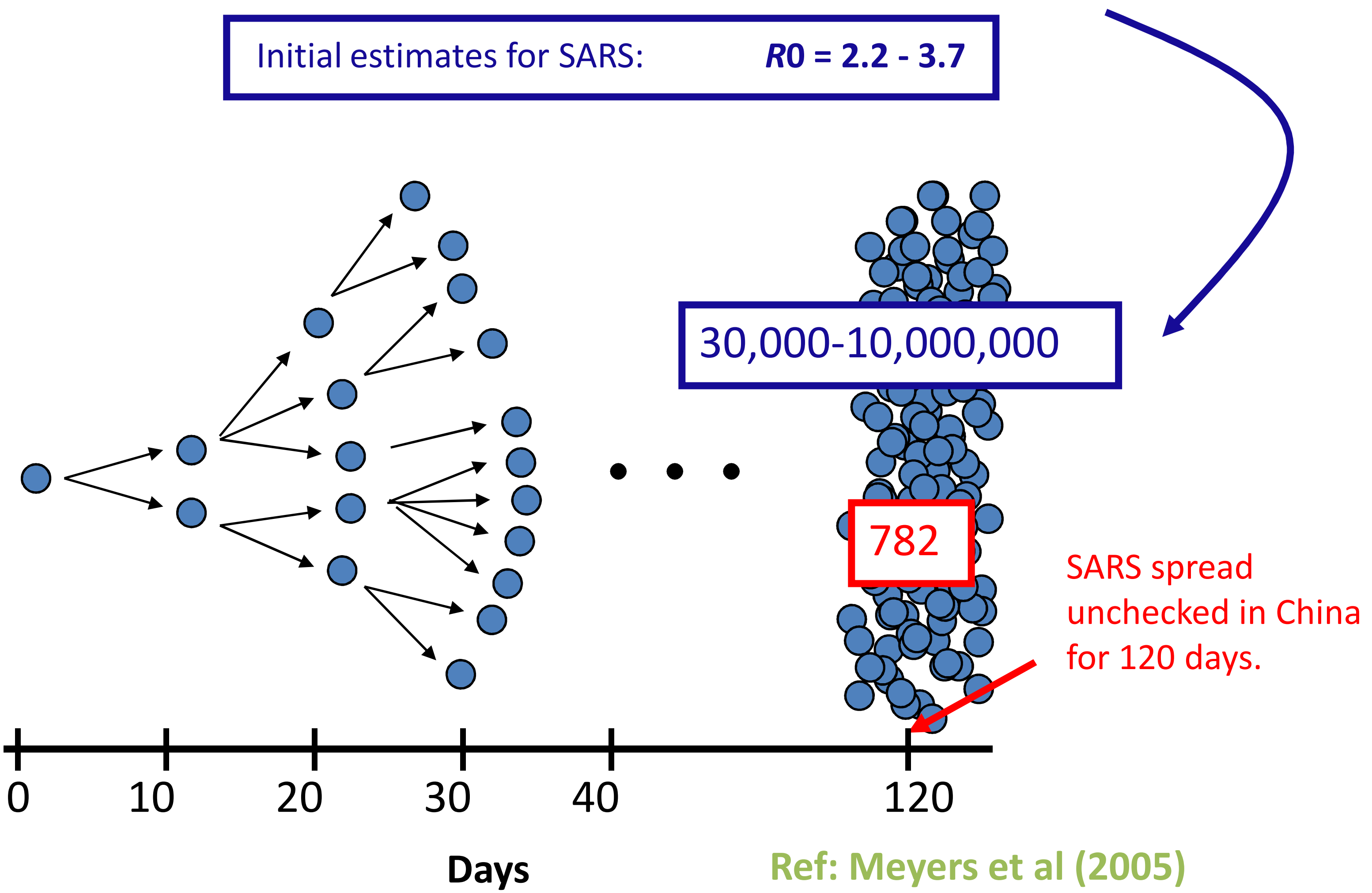


$$R0 = \frac{\text{Infection rate}}{\text{Mortality + Recovery rate}}$$

Reproductive Ratio/Number: average number of secondary cases per infected individual

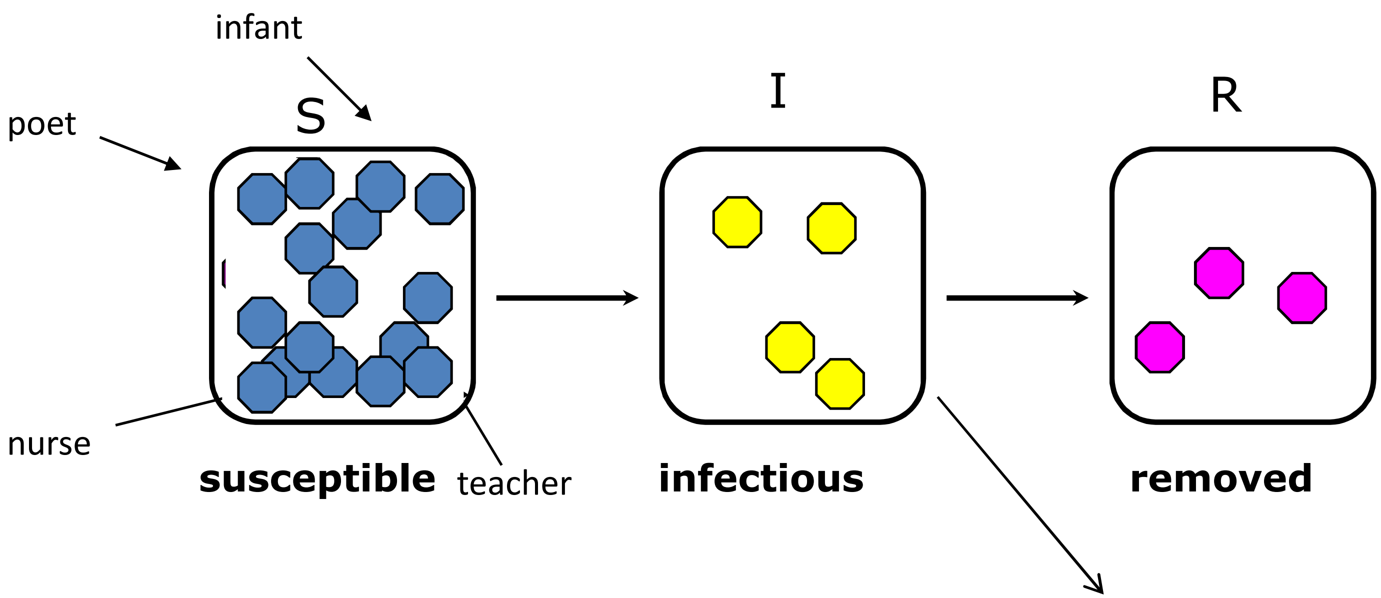
SARS and its Reproductive Ratio (R0)

R_0= reproductive ratio → number of secondary infections caused by a single infected person

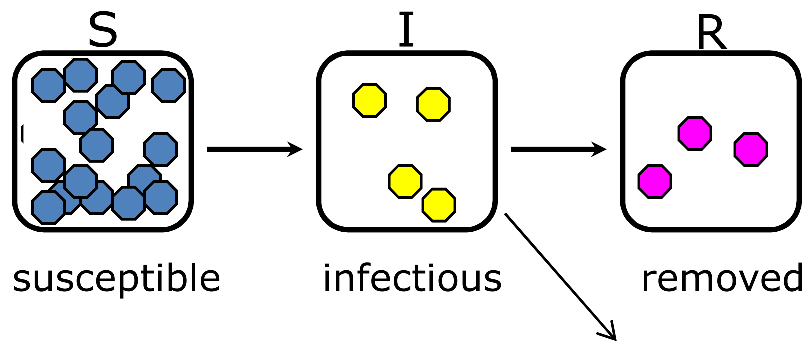


Ref: Meyers et al (2005)
J Theor Biol. 2005 Jan 7;232(1):71-81

What's missing in this model?



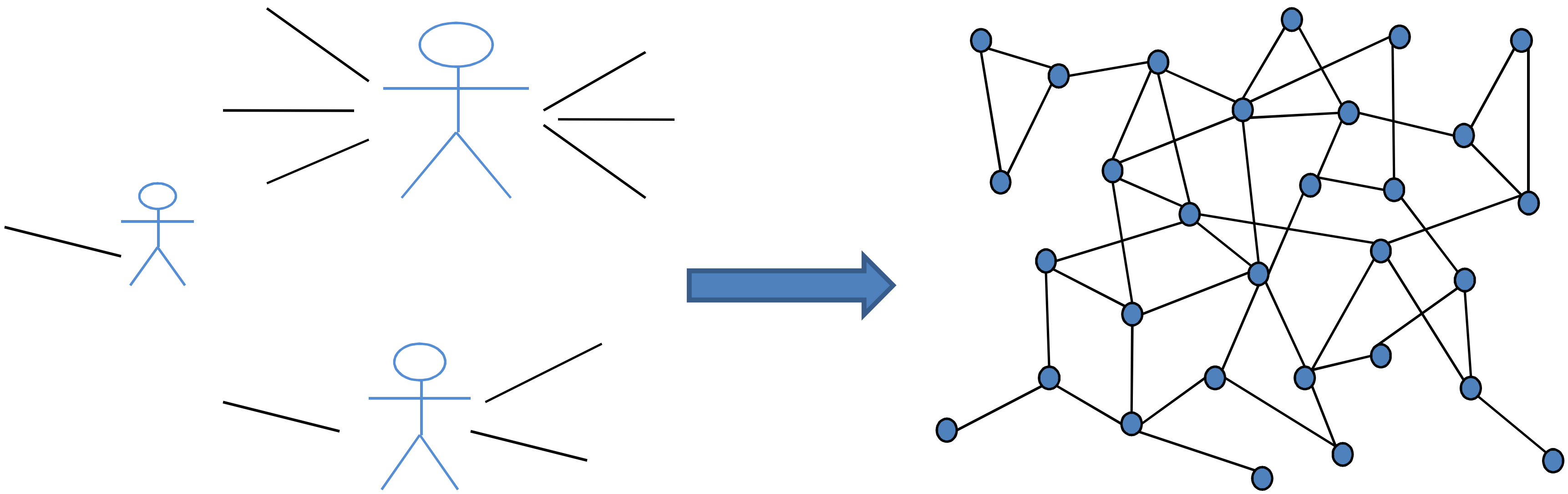
Review: Compartmental Models



Advantages:
- Simple
-Extendable
- Amenable to mathematical analysis

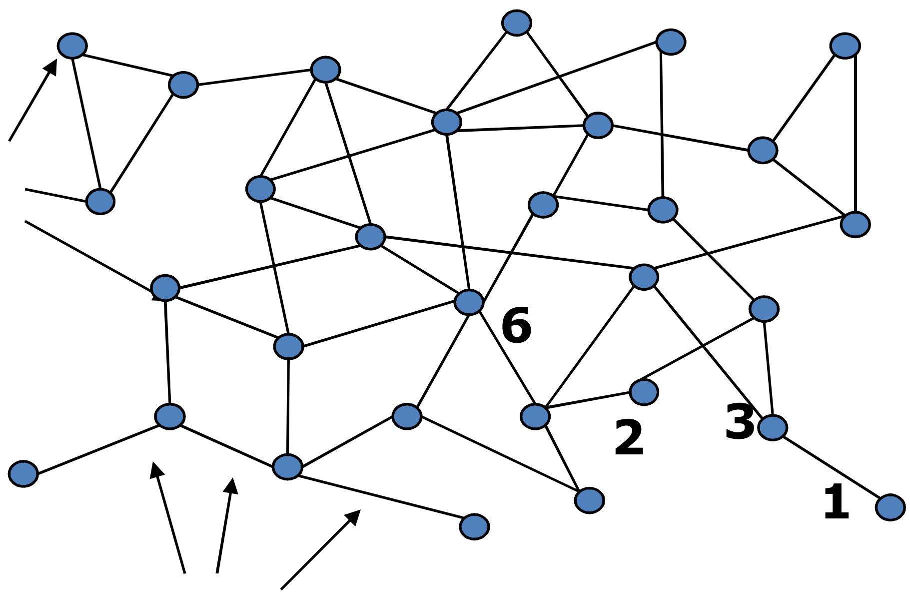
Disadvantage:
- Assume everyone in the population is equally vulnerable to infection and to spreading infection.

What can we do to fix this?



Terminology

Node (vertex): These represent people or places that can become infected

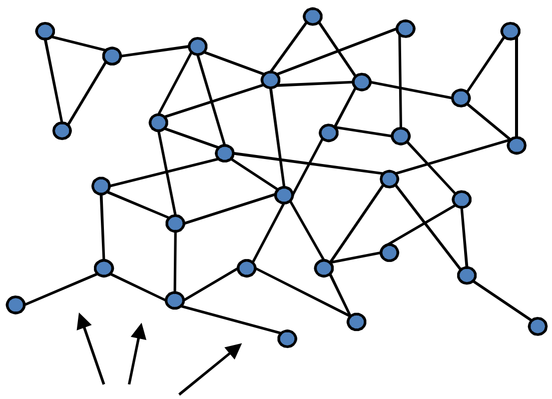


Degree: The number of edges coming out of a node

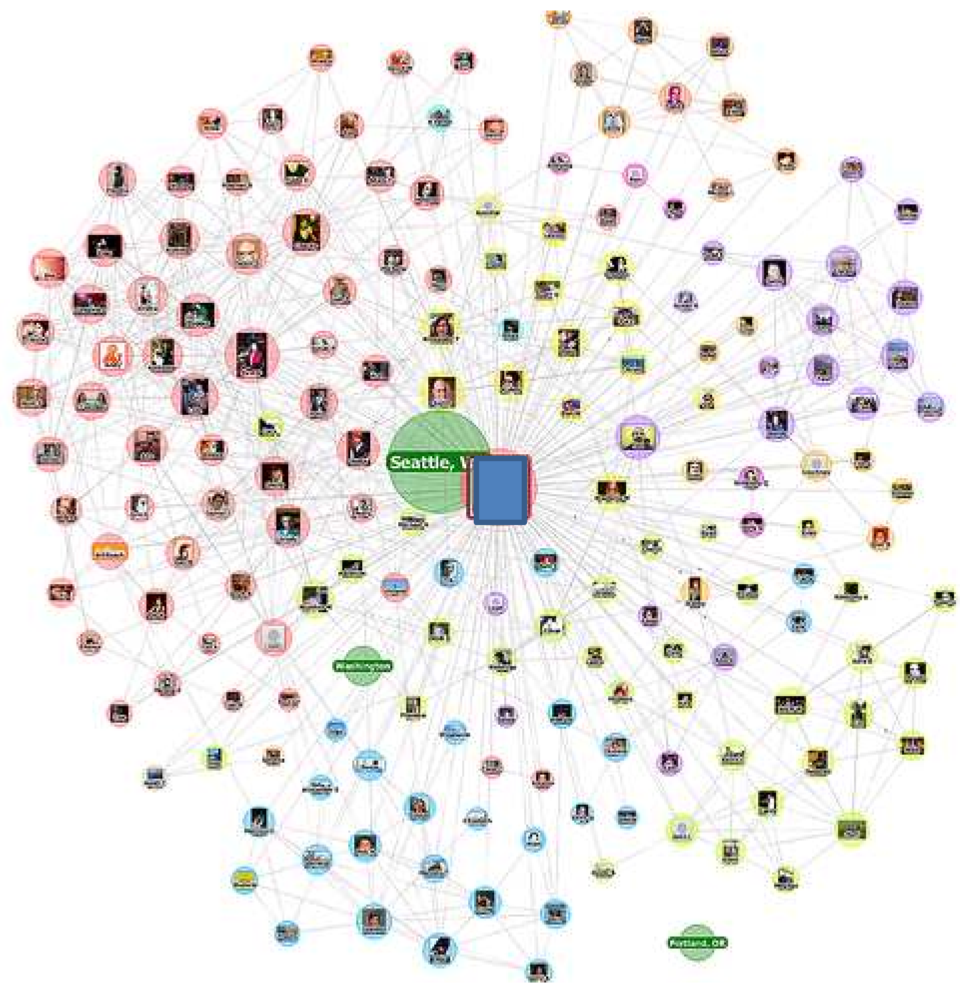
Edge: contacts between nodes (people) that can lead to disease transmission

CONTACT NETWORK (graph)

What makes up a contact network?

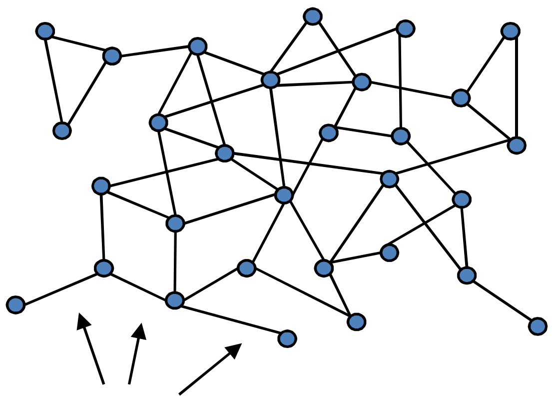


Edge: contacts between nodes (people) that can lead to disease transmission



Social Network ≠ Contact Network

What makes up a contact network?



Edge: contacts between nodes (people) that can lead to disease transmission

"Disease-causing contact":

~~Talking on phone~~
~~Texting~~
~~Emailing~~

Shaking hands
Touching door knobs
Sneezing on

Respiratory diseases

Sexual contact
Needle sharing

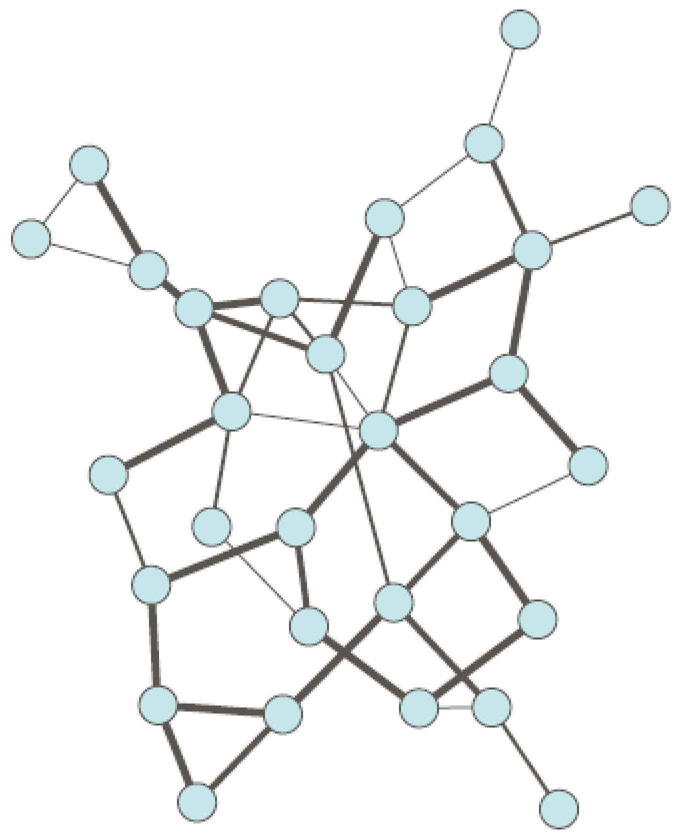
Sexually-transmitted diseases

Contact Network: specific to a population and a disease class

Capturing Contacts in Networks

Weighted Contacts

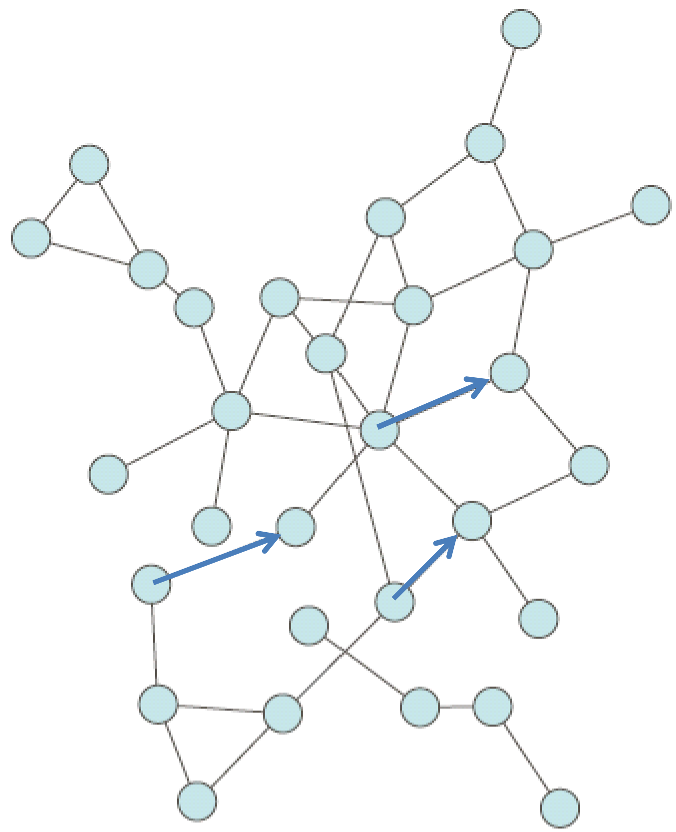
A way to represent the strength of a contact



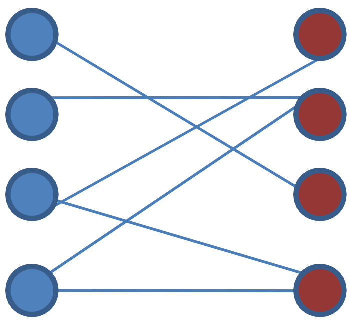
E.g.
Duration of contact,
Number of passengers

Directed Contacts

A way to represent asymmetry in contact

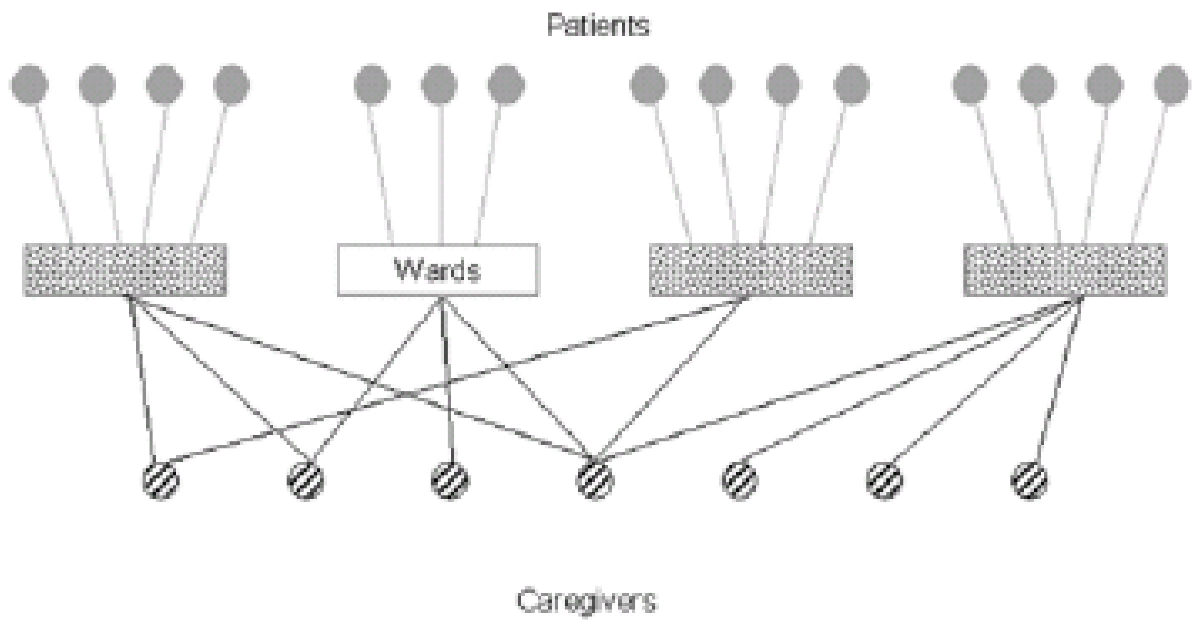


Bipartite Networks



Transmission of STD in heterosexual network

Co-location Network



Ref: Meyers et al (2003)
Emerging Infectious Diseases 9, 204-210

Office Hours

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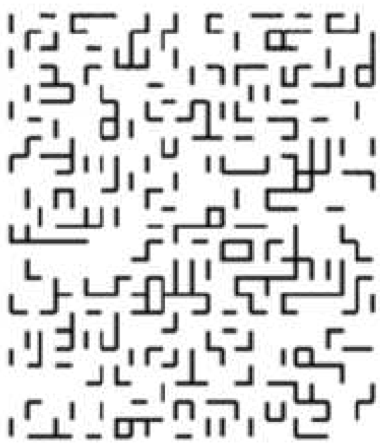
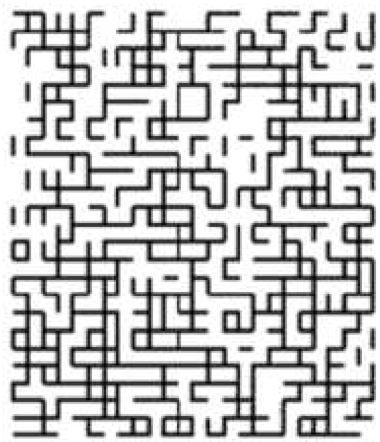
Predicting Epidemics

Bond percolation

- Start with a lattice (or network)
- Draw (or mark) the edges with a certain probability *p*
- The remaining edges are open (unmarked)
- At a critical probability p_c a spanning cluster appears



p=0.315

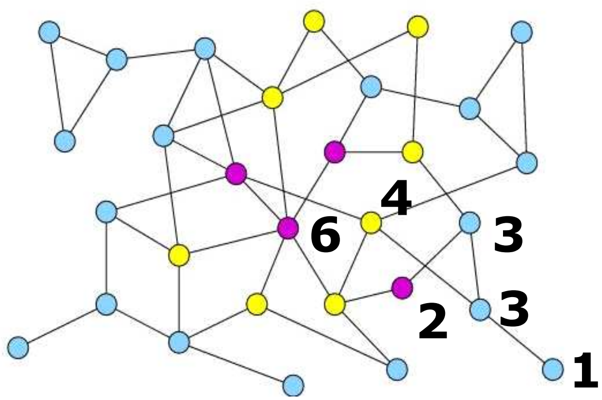


p=0.525

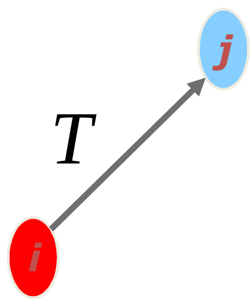
Ref: H. L Frisch and J. M. Hammersley, J. SIAM 11, 894 (1963)

Predicting Epidemics

Degree Distribution



Transmissibility



Probability of transmission:

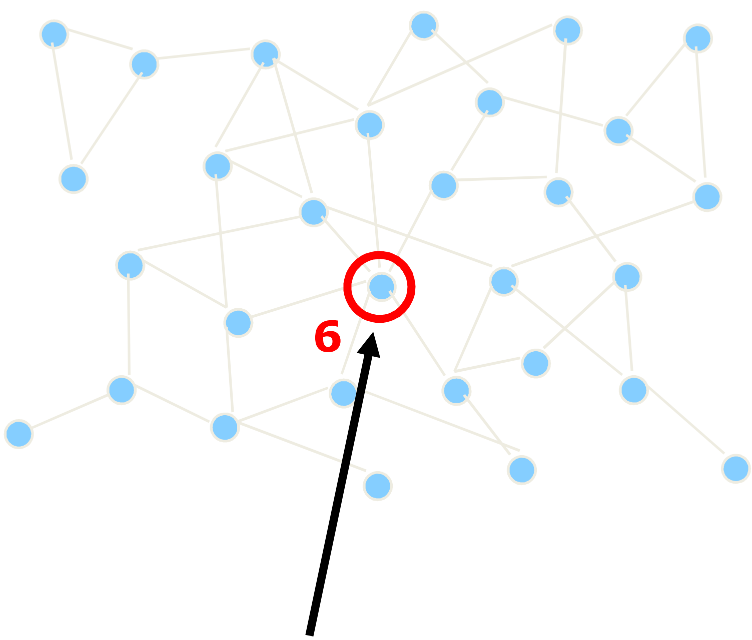
- rate of contact
- infectiousness
- susceptibility

Predicting epidemics: Percolation

Probability generating function (PGF) for the degree distribution

$$G_0(x) = \sum_{k=0}^{\infty} p_k x^k$$

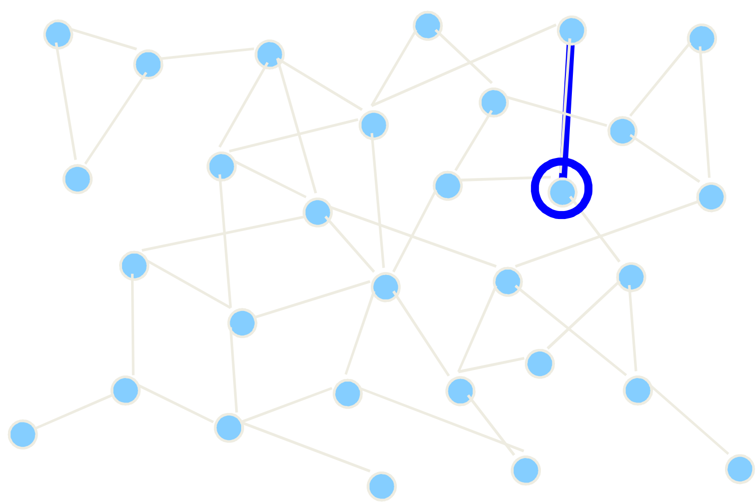
Degree	Frequency
1	3/30 = 0.1
2	7/30 = 0.23
3	11/30 = .37
4	6/30 = 0.2
5	2/30 = 0.07
6	1/30 = 0.03



G_0 tells you about the degrees of randomly chosen vertices

Ref: H. Wilf, Generatingfunctionology (1994)

Predicting epidemics: Percolation



G_1 tells you about the degrees of vertices arrived at along randomly chosen edges

$$G_1(x) = \frac{1}{\langle k \rangle} G_0'(x)$$

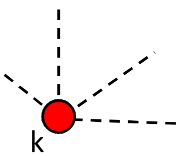
PGF for the <u>excess</u> degree distribution

Ref: Newman (2002) Phys. Rev. E 66, 016128 (2002)

Predicting epidemics: Percolation

Including disease transmission

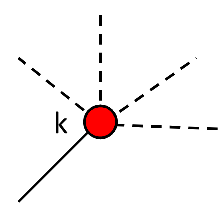
The pgf for the number of secondary infections leading from an infected vertex



$$G_0(x;T) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} p_k \underbrace{\binom{k}{m} T^m (1-T)^{k-m}}_{\text{the probability } m \text{ of the } k \text{ edges transmit infection}} x^m = G_0(1+(x-1)T)$$

the probability that a randomly selected vertex has degree *k*

$$G_1(x;T) = G_1(1+(x-1)T)$$



Predicting epidemics: Percolation

$H_0(x;T)$ pgf for infected cluster size distribution

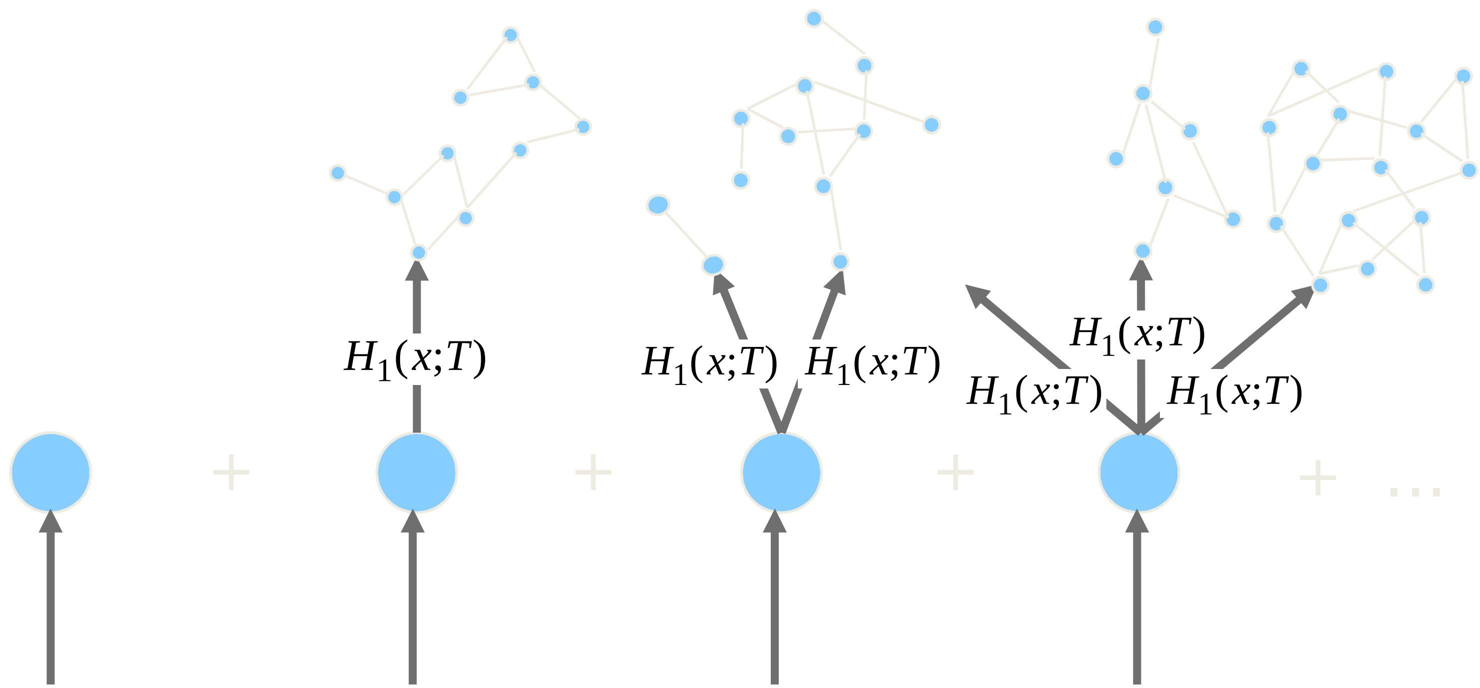
$H_1(x;T)$ pgf for infected cluster size distribution at end of randomly chosen edge

Ref: Newman (2002) Phys. Rev. E 66, 016128 (2002)

Predicting epidemics: Percolation

The pgf for the cluster size distribution at the end of a randomly chosen edge:

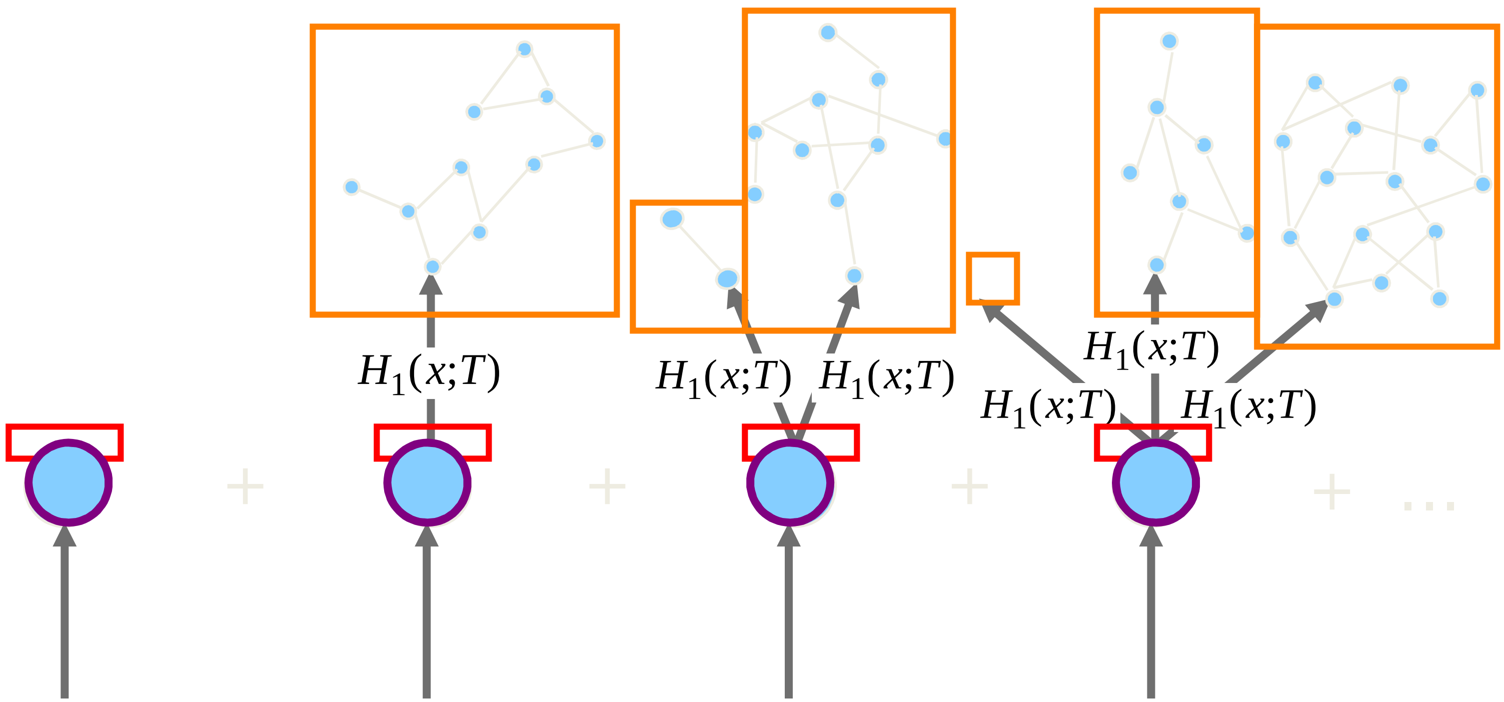
$$H_1(x;T)$$



Predicting epidemics: Percolation

The pgf for the cluster size distribution at the end of a randomly chosen edge:

$$H_1(x;T) = x G_1(H_1(x;T);T)$$



Predicting Epidemics: the Math

$H_0(x;T)$ pgf for infected cluster size distribution

$H_0'(1;T) = \langle s \rangle$ average infected cluster size

$$\langle s \rangle = 1 + \frac{G_0'(1;T)}{1 - G_1'(1;T)} = 1 + \frac{TG_0'(1)}{\boxed{1 - TG_1'(1)}}$$

$$T_c = 1 / G_1'(1)$$ Epidemic threshold

Ref: Newman (2002) Phys. Rev. E 66, 016128 (2002)

Predicting Epidemics: Epidemic Threshold

$T < T_c$

only small disease outbreaks occur whose avg. size will be <s>

$T > T_c$

larger epidemics can occur, and <s> does not apply

Predicting Epidemics: Epidemic Size

$T > T_c$ **Epidemic possible**

What will be the probability of an epidemic?

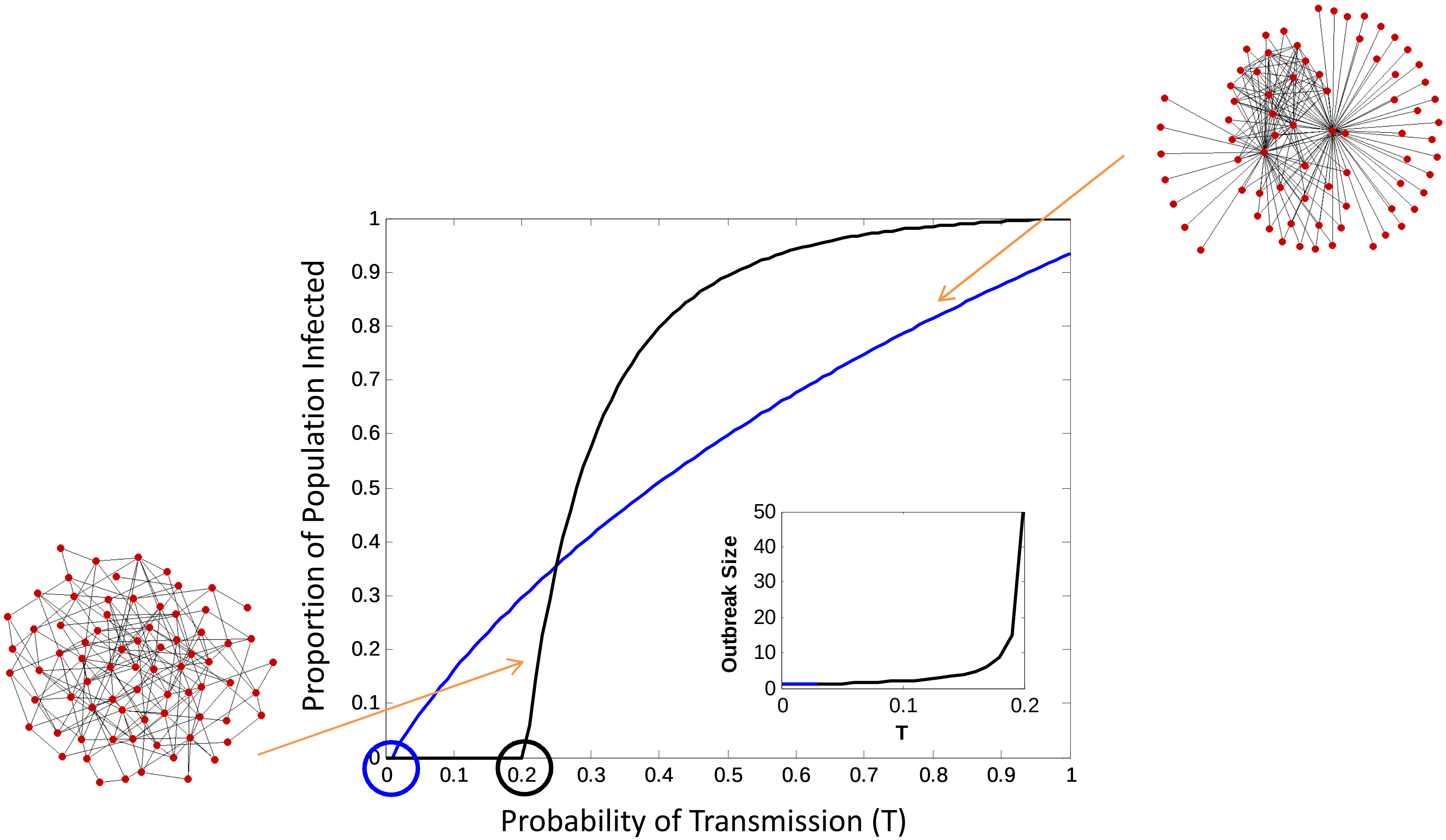
***P* = 1 – Prob(only small outbreaks)**

$= 1 - H_0(1;T)$

$\boldsymbol{S} = 1 - H_0(1;T)$

Size of epidemic

Predicting Epidemics: Results



Dynamical Models on Networks

- Pair approximation methods

  Ref: Keeling (1999) Proc. Roy. Soc. Lond. B 266 859-869

- Heterogeneous-mixing methods

  Ref: Pastor-Satorras &Vespignani (2002) Phys. Rev. E 65, 035108

- Dynamical PGF methods

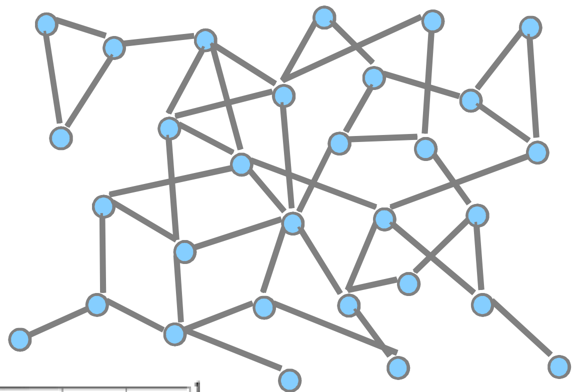
  Ref: Volz (2008) Journal of Mathematical Biology, 56 3

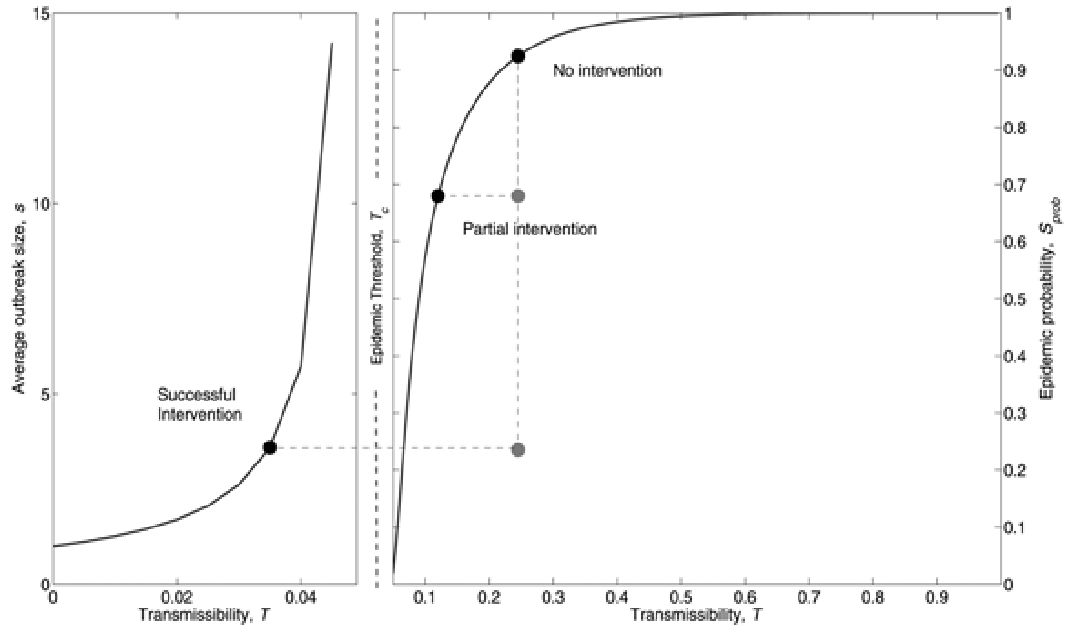
Controlling Epidemics

Assessing Control Strategies (I)

- Transmission-reducing interventions (reducing T_{ij} on some or all edges)

- Face masks, gloves
- Washing hands

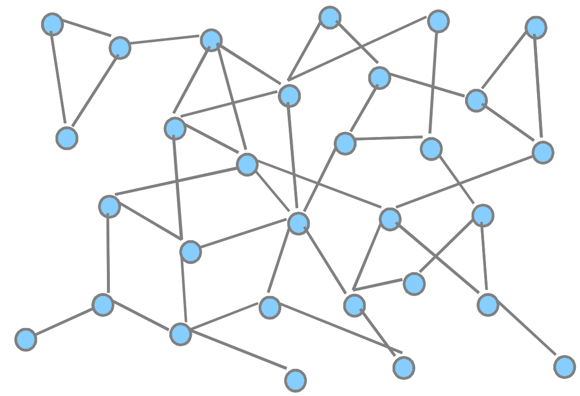


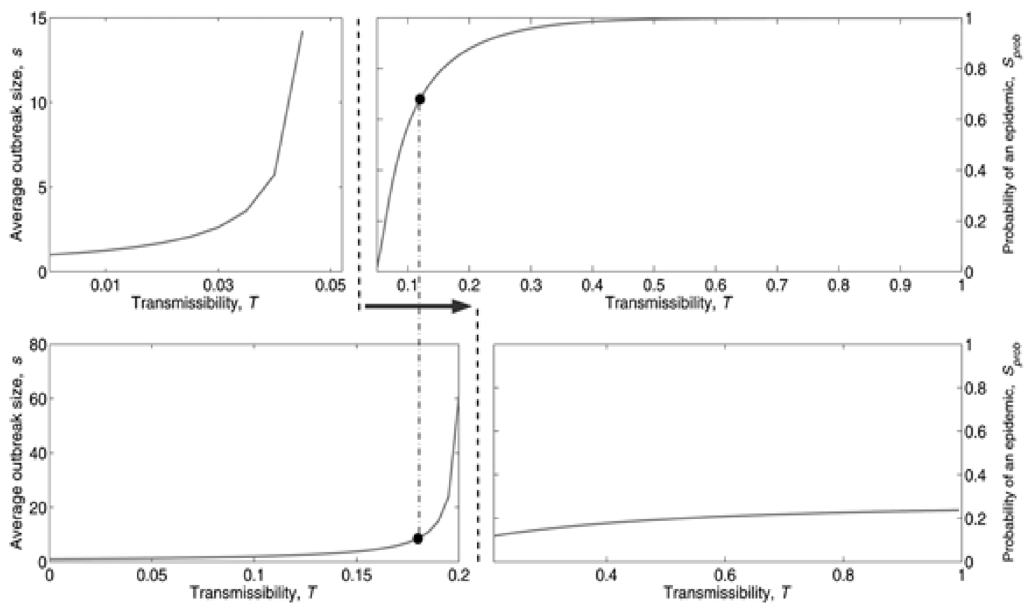


Ref: Pourbohloul, Meyers, et al. (2005) EID Vol 11

Assessing Control Strategies (II)

- Contact-reducing interventions (reducing number of edges)
  - Quarantining a patient
  - Closing schools
  - Social distancing

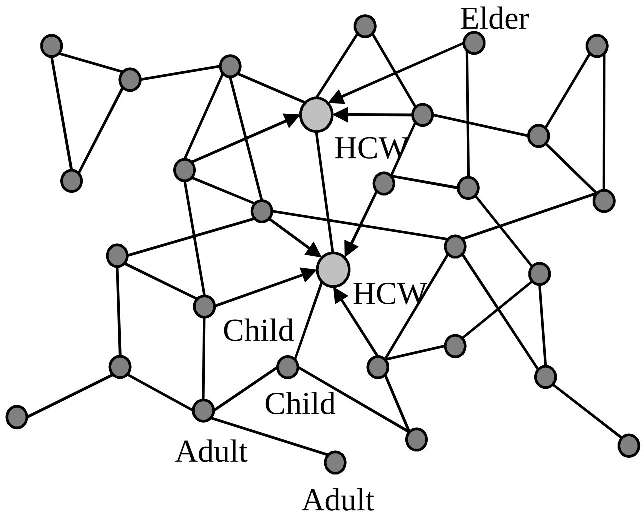




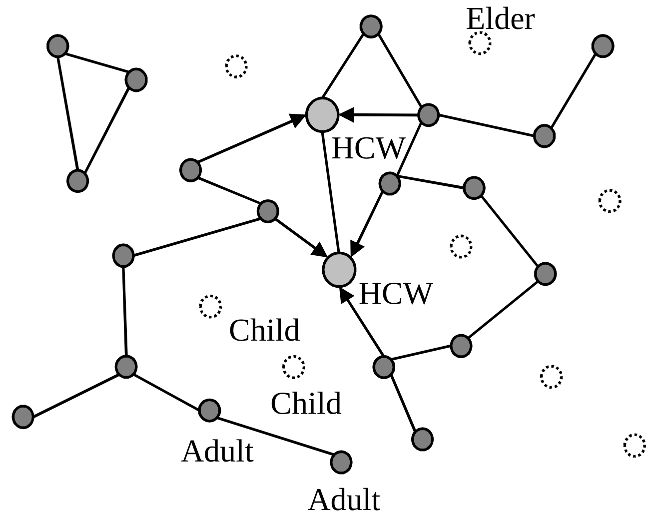
Ref: Pourbohloul, Meyers, et al. (2005) EID Vol 11

Assessing Control Strategies (III)

- Immunization



Total Population



Vaccinated Population